itute.com

= **Year 12** = **Algebra of functions** = **Worksheet 3**

Given the rules $f(x)$ and $g(x)$, find the rules $f(g(x))$ and $g(f(x))$, state the domain and range in each case. 1. $f(x)=\sqrt{x}$; $g(x)=x^2$.	2. $f(x)=\sin x$ where $x\in[0,2\pi]$; $g(x)=\sqrt{x}$.
3. $f(x)=x^2+1$; $g(x)=\log_e x$.	4. $f(x)=\lvert x\rvert$; $g(x)=\cos x$.
5. $f(x)=\tan x$ where $x\in\left[0,\frac{\pi}{2}\right)$; $g(x)=x^2+1$.	6. $f(x)=e^x$; $g(x)=x^2-4x-5$.
7. $f(x)=\frac{1}{x+1}$; $g(x)=x^2$.	8. $f(x)=\sin x$; $g(x)=1-e^x$.
9. $f(x)=\frac{1}{x}$; $g(x)=\cos x$ where $x\in[0,2\pi]$.	10. $f(x)=1+\frac{1}{x^2}$; $g(x)=-\frac{1}{\sqrt{x-1}}$.

Numerical, algebraic and worded answers. 5. $f(g(x))=\tan(x^2+1)$, $\left[0,\sqrt{\frac{\pi}{2}-1}\right)$, $[\tan 1,\infty)$; $g(f(x))=\tan^2 x+1$, $\left[0,\frac{\pi}{2}\right)$, $[1,\infty)$.

3. $f(g(x))=(\log_e x)^2+1$, R^+, $[1,\infty)$; $g(f(x))=\log_e(x^2+1)$, R, $[0,\infty)$. 4. $f(g(x))=\lvert\cos x\rvert$, R, $[0,1]$; $g(f(x))=\cos\lvert x\rvert$, R, $[-1,1]$.

1. $f(g(x))=\lvert x\rvert$, R, $[0,\infty)$; $g(f(x))=x$, $[0,\infty)$, $[0,\infty)$. 2. $f(g(x))=\sin\sqrt{x}$, $[0,\infty)$, $[-1,1]$; $g(f(x))=\sqrt{\sin x}$, $[0,\pi]$, $[0,1]$.

6. $f(g(x))=e^{x^2-4x-5}$, R, $(0,\infty)$; $g(f(x))=e^{2x}-4e^x-5$, R, $[-9,\infty)$. 7. $f(g(x))=\frac{1}{x^2+1}$, R, $(0,1]$; $g(f(x))=\frac{1}{(x+1)^2}$, $R\setminus\{-1\}$, R^+

8. $f(g(x))=\sin(1-e^x)$, R, $[-1,1]$; $g(f(x))=1-e^{\sin x}$, R, $[1-e,1-e^{-1}]$. 10. $f(g(x))=x$, $(1,\infty)$, $(1,\infty)$; $g(f(x))=-x$, $R\setminus\{0\}$, $R\setminus\{0\}$.

9. $f(g(x))=\frac{1}{\cos x}$, $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right]$, $(-\infty,-1]\cup[1,\infty)$; $g(f(x))=\cos\frac{1}{x}$, $R\setminus\{0\}$, $[-1,1]$.