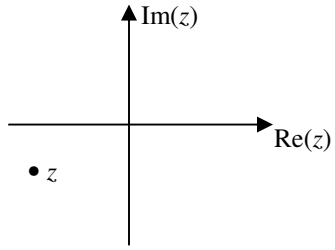
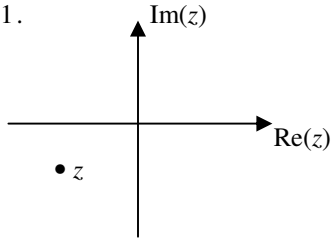
= Year 12 = Complex numbers = Worksheet 2

You need a ruler and/or a protractor to do Q1 to 4. Both axes have the same scale.

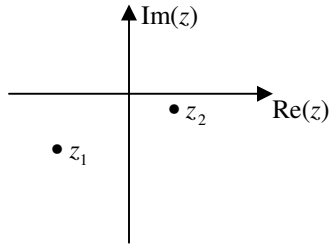
1. z is shown in the argand diagram. Plot iz, $-z$ and $-iz$.



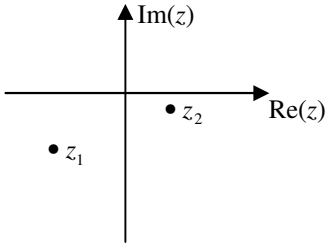
2. $z = 2cis\theta$ is shown in the argand diagram. Plot $\bar{z}$, z^{-1} and $z+1$.



3. z_1 and z_2 are shown in the argand diagram.
Plot $z_1 + z_2$, $z_1 - z_2$ and $2z_1 + z_2$.



4. $z_1 = 2cis\alpha$ and $z_2 = cis\beta$ are shown in the argand diagram.
Plot $z_1 z_2$, z_2 / z_1 and $z_2^{\,3}$.



5. Simplify $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^5$.

6. Simplify $\frac{\left(\sqrt{3}-i\right)^3}{(1+i)^6}$.

7. Find the cube roots of -8 in $x + yi$ form.

8. Find z such that $z^{\frac{3}{2}} = -8$. Express answers in $x + yi$ form.

9. Simplify $\left(\frac{z+\bar{z}}{2}\right)^2 - \left(\frac{z-\bar{z}}{2}\right)^2$.

10. Simplify (i) $(z+iz)(\bar{z}-i\bar{z})$ and (ii) $(z+iz)(\bar{z}+i\bar{z})$.

11. Given $z_1 = 2cis\alpha$ and $z_2 = cis\beta$, find $|z_1 - z_2|^2$ in terms of α and β.

Numerical, algebraic and worded answers.

5. $1/\sqrt{2} - (1/\sqrt{2})i$
6. 1
7. $1+i\sqrt{3}$, $-2+0i$, $1-i\sqrt{3}$
8. $4+0i$, $-2+i2\sqrt{3}$, $-2-i2\sqrt{3}$
9. $|z|^2$
10. (i) $2|z|^2$ (ii) $2|z|^2 i$
11. $5 - 4\cos(\alpha-\beta)$