2016 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core
Data analysis

Q1ai 17.8

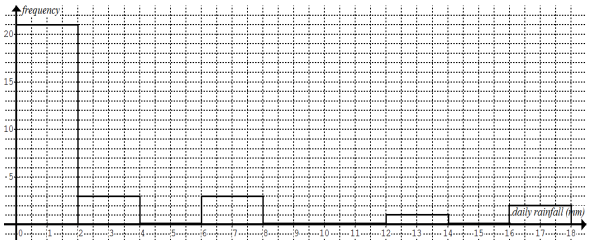
Q1aii 0

Q1b The dot at 2.6 is Q_3

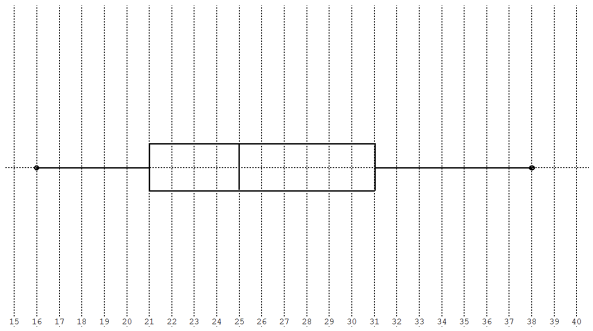
Q1ci 16 days

Q1cii $\frac{3}{30} = 10\%$

Q1d



Q2ai



Q2aii 75%

Q2bi
July: Positively skewed with an outlier at the high end

May: Approximately symmetric with no outliers

Q2bii Upper fence $= Q_3 + 1.5 \times IQR = 11 + 1.5 \times 3 = 15.5$

Q2biii The median maximum daily temperatures for the two given months are very different, about 14.4°C for May and 9.1°C for July, .: the maximum daily temperature is associated with the month of the year.

Q3a The association is trong, positive and linear.

Q3bi Using the line shown in the scatterplot, the slope is calculated to be 0.95 and the intercept -1.8.
$apparent\ temperature = -1.8 + 0.95 \times actual\ temperature$

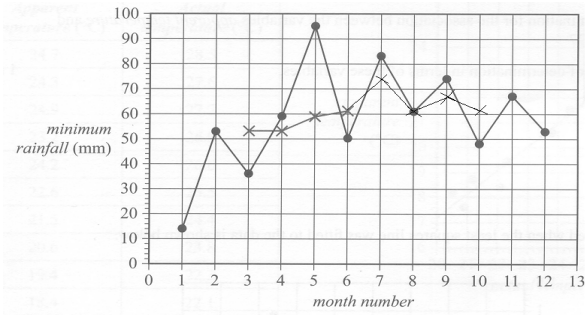
Q3bii $apparent\ temperature - 0.95 \times actual\ temperature = -1.8$
$\therefore\ apparent\ temperature - actual\ temperature \approx -1.8$
i.e. the apparent temperature is about 1.8°C lower than the actual temperature.

Q3c About 97% of the variability of the apparent temperature is explained by the variability of the actual temperature using the linear regression model.

Q3di It is assumed that the association is linear between the apparent temperature and the actual temperature.

Q3dii Yes, it is a random pattern.

Q4a



Q4b The mean of Sep. and Oct. $= \frac{124 + 140}{2} = 132$

The mean of Oct. and Nov. $= \frac{140 + 225}{2} = 182.5$

The 2-mean smoothed rainfall centred on Oct.
$= \frac{132 + 182.5}{2} = 157.25\ \text{mm}$

Recursion and financial modelling

Q5a Initial deposit $= V_0 = \$15000$ (Note: the unit for the amount was not given in part a)

Q5b $V_1 = 1.04 \times \$15000 = \$15600$, $V_2 = 1.04 \times \$15600 = \$16224$

Q5c 4%

Q5di $V_n = 1.04^n \times 15000$

Q5dii $V_{10} = 1.04^{10} \times 15000 = \$22203.66$

Q6a Average depreciation per year $= \frac{38000 - 16000}{8} = \$2750$

Q6b $C_{n+1} = C_n - 2750$

Q6c Amount depreciated in 8 years $= 38000 - 16000 = 22000$
Depreciation $= \frac{22000}{5000 \times 8} = \$0.55$ per kilometre travelled

Q7ai (In the past exams the term 'reducing balance loan' was specified in the questions. In Sample Exam 2 the term was not specified in the question and the loan was calculated as non-reducing. In this question 'to reduce the balance' is used in part b, so the loan will be treated as a reducing balance loan.)

$N = 12, I\% = 6.9, PV = 70000, PMT = -800, P/Y = 12, C/Y = 12$
$FV = -65076.22$
Amount owing after 12 payments $= \$65076.22$

Q7aii Total interest paid after 12 payments
$= 800 \times 12 - (70000 - 65076.22) = \$4676.22$

Q7b After the first 3 years, $FV = -54151.60$
After paying a lump sum of $\$L$, the balance is PV to be repaid in 3 years.
$N = 36, I = 6.9\%, PMT = -800, FV = 0, P/Y = 12, C/Y = 12$
$PV = 25947.58$
$\$L = 54151.60 - 25947.58 \approx \$28204$

SECTION B - Modules

Module 1: Matrices

Q1a 4×1

Q1bi $J = G \times C = \begin{bmatrix} 40 & 25 & 15 & 30 \end{bmatrix} \begin{bmatrix} 85 \\ 38 \\ 24 \\ 43 \end{bmatrix} = \begin{bmatrix} 6000 \end{bmatrix}$

Q2a Ben and Elka

Q2b Amara and Dana

Q3a $d = 298$, $e = 94$, $f = 130$

Q3b $0.65 \times 520 + 0.25 \times 320 + 0.25 \times 80 + 0.50 \times 80 = 478$

Q3c $0.25 \times 80 = 20$

Q3d $\dfrac{0.65 \times 520}{478} \approx 0.7071 \approx 71\%$

Q3ei 80 customers are removed from the company customer list because they have not made booking with the company for a long time.

Q3eii $R_{2017} = TR_{2016} + B = \begin{bmatrix} 699.65 \\ 501.45 \\ 176.8 \\ 102.1 \end{bmatrix}$

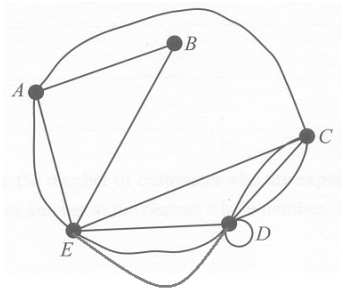
$R_{2018} = TR_{2017} + B = \begin{bmatrix} 755.385 \\ 536.4925 \\ 189.7475 \\ 118.375 \end{bmatrix} \approx \begin{bmatrix} 755 \\ 536 \\ 190 \\ 118 \end{bmatrix}$ (Rounding errors exist)

190 customers are expected to choose sea travel in 2018.

Module 2: Networks and decision mathematics

Q1a Alooma and Easyside

Q1bi



Q1bii The loop at D represents a route that a driver can depart from Dovenest and return to Dovenest without passing through another suburb, and without turning back.

Q2a Eulerian trail *XYTUYZUVZWV* .: Ramp *V*

Q2b A Hamiltonian path *XYTUZVW*

Q2c *XYTUZVWX*, *XYTUVZWX* or in reverse order for both, .: 4 ways

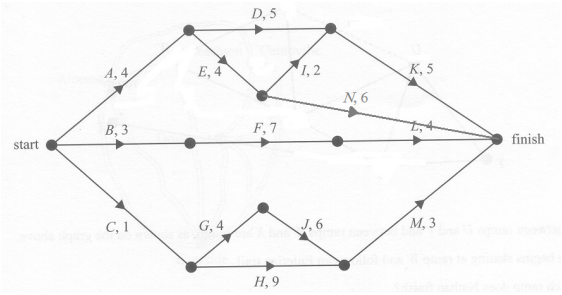
Q3a Earliest start time for activity $M = 1 + 4 + 6 = 11$ days from start

Q3b *AEIK*

Q3c *H*

Q3d The shortest time possible is 14 days after reduction by one day for activity *E* or *I* on the critical path. Reducing *I* by one day will incur the least cost of $2000.

Q3ei



Q3eii Latest start time for activity $N = 9$ days from start

Module 3: Geometry and measurement

Q1a $\text{Area} = 4\times\pi\times 21.4^2 \approx 5755 \text{ mm}^2$

Q1b $\text{Minimum length} = 21.4\times 10 = 214 \text{ mm}$

Q2a $\text{Distance apart} = 50\sin 30° = 25 \text{ m}$

Q2b $\text{Angle of elevation} = \tan^{-1}\left(\frac{16.8}{200}\right) \approx 5°$

Q3a $\text{Shortest great circle distance} = \frac{56}{360}\times 2\times\pi\times 6400 \approx 6255 \text{ km}$

Q3b Add 10 hours to 6:32 am. The tournament will begin in Melbourne on Thursday 4:32 pm.

Q4a Distance $PR = \sqrt{80^2+100^2-2(80)(100)\cos 104°} \approx 142 \text{ m}$

Q4b $\frac{\sin\angle RPQ}{100} = \frac{\sin 104°}{142}$, $\angle RPQ \approx 43°$

Bearing of R from $P = 130 - 43 = 087°$

Q5a $\frac{100}{360}\times\pi d^2 = 147.5$, $d \approx 13 \text{ m}$

Q5b $\text{Area} = \frac{360-100}{360}\left(\pi\times 12^2 - \pi\times 7.5^2\right) \approx 199 \text{ m}^2$

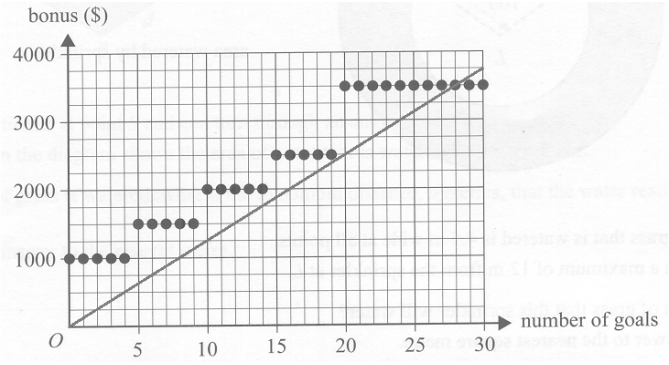
Module 4: Graphs and relations

Q1a \$1500

Q1b 15 goals

Q1c $\$125\times 8 = \$1000$

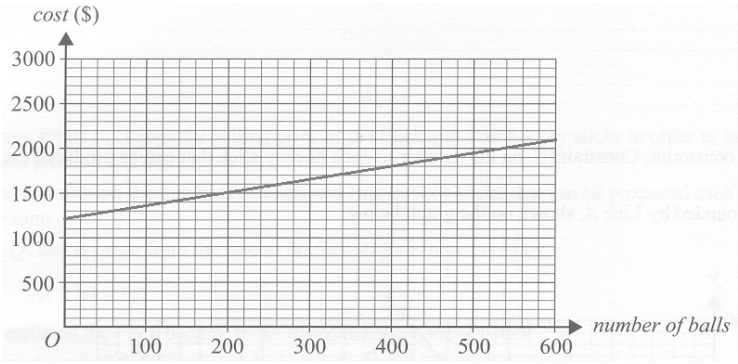
Q1d The graph of bonus vs number of goals for Bianca shown below has a slope of \$125 per goal.



Both players scored 28 goals to receive the same bonus amount of \$3500.

Q2a $1650 = 1200 + 1.5\times n$, $n = 300$

Q2b



Q2c Let $\$p$ be the selling price of one ball.

$200p = 1200 + 1.5\times 200$, $200p = 1500$, $p = 7.50$

Q3a At most two Jink sticks are produced for each Flick stick produced.

Q3b $y \le 300$

Q3c $(200, 300)$ will produce maximum profit.

$P_{\max} = 62\times 200 + 86\times 300 = \$38200$

Q3d The profit line $mx + ny = 42000$ and constraint line $x + y = 500$ are the same. $\therefore\ m = n$ and $400m + 100n = 42000$

$\therefore\ 500m = 42000$, $m = n = 84$

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