2022 VCAA Further Mathematics Exam 2 Solutions

SECTION A - Core

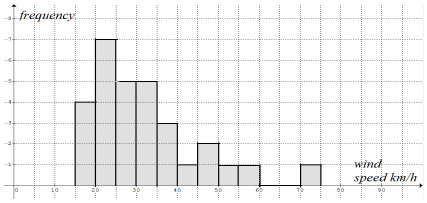
Data analysis

Q1ai 20 Q1aii $\frac{18}{30} = 0.6 = 60\%$

Q1b $IQR = 35 - 22 = 13$, upper $35 + 1.5 \times 13 = 54.5$

$59, 70 > 54.5$.: they are outliers

Q1c



Q2ai More variable, $IQR(9am) = 35$ is greater than $IQR(3pm) = 28$

Q2aii 75%

Q2aiii

40			100	100
			80	100

Q2b Yes, there is an association between relative humidity and time of day. The morning median 75 is greater than the afternoon median.

Q3ai Six numerical variables.
Day number is time and has equal interval between two consecutive days. It is considered as a numerical variable. A numerical relationship (if it exists) can be established between a response variable and variable *day number*.

Q3aii Median is 0.

Q3b $maximum\ temperature = 9.236 + 1.002 \times minimum\ temperature$

Q3c 61.1%

Q4a Temperature

Q4b $r = -0.78$, the association is moderately strong and trending negatively.

Q4c On average for each degree increase in temperature there is a 3.417% decrease in relative humidity.

Q4d $predicted\ value = 120.1 - 3.417 \times 16.3 \approx 64.40$, $actual = 45.0$
$residual \approx 45.0 - 64.40 \approx -19.4$

Q5a The pattern shows seasonality and negative trend.

Q5b 100

Q5ci
$\frac{129.6 + 179.6 + 206.2 + 155.2 + 127.6 + 83.8 + 61.8 + 134.6 + 122.8}{9} \approx 133.5$

Q5cii $36 - 4 - 4 = 28$

Recursion and financial modelling

Q6a $V_1 = 200000 - 12500 = 187500$

Q6b $200000 - 12500n = 0$, $n = 16$

Q6c $V_0 = 200000$, $V_{n+1} = V_n - 12500$

Q6d Deducing balance depreciation

Q7a \$5214.28

Q7b $\$532261.78 \times \frac{0.03}{12} = \$532261.78 \times 0.0025 \approx \$1330.65$

Q7c $P_0 = 540000$, $P_{n+1} = 1.0025P_n - 5214.28$

Q7d $\frac{\text{simple interest}}{12} = \$1350.00$

Q8a $L_1 = 1.002 \times 580000 - 3045.26 = 578114.74$
$L_2 = 1.002 \times 578114.74 - 3045.26 = 576225.71$

Q8b $0.002 \times 12 = 0.024 = 2.4\%$

Q8c TVM Solver: N = 239.9999509
Let N = 240, giving FV = 0.15 .: the small amount is \$0.15

Q8d Let N = 240 − 12 = 228 and FV = 0, giving I% = 1.9466%
.: monthly $= \frac{1.9466\%}{12} \approx 0.16222\% \approx 0.0016222$
.: multiplication factor is 1.0016 (4 decimal places)

SECTION B - Modules

Module 1: Matrices

Q1a $[5] \times [80 \quad 140 \quad 270] = [400 \quad 700 \quad 1350]$

Q1b $A = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

Q2a $\begin{bmatrix} 20 & 40 \\ 30 & 50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3700 \\ 4900 \end{bmatrix}$

Q2bi $20 \times 50 - 30 \times 40 = -200$

Q2bii Since the determinant is non-zero, .: the simultaneous linear equations have a unique solution set, one *x* value and one *y* value.

Q2c $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 30 & 50 \end{bmatrix}^{-1} \begin{bmatrix} 3700 \\ 4900 \end{bmatrix}$.: $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 55 \\ 65 \end{bmatrix}$

Q3a $0.7 \times 210 + 0.7 \times 190 + 0.8 \times 80 = 344$

Q3b $\frac{0.8 \times 80}{0 \times 210 + 0.2 \times 190 + 0.8 \times 80} \approx 0.62745 = 63\%$

Q3c Repeat application of T: $S_1 = \begin{bmatrix} 166 \\ 212 \\ 102 \end{bmatrix}$, $S_2 = \begin{bmatrix} 137.4 \\ 218.6 \\ 124 \end{bmatrix}$,

$S_3 = \begin{bmatrix} 118.04 \\ 219.04 \\ 142.92 \end{bmatrix}$, $S_4 = \begin{bmatrix} 104.532 \\ 217.324 \\ 158.144 \end{bmatrix}$, …

intermediate starts to decrease after S_3. Max. number = 219

Q4a $\begin{bmatrix}0.4 & 0.2 & 0.3\\0.3 & 0.5 & 0.2\\0.3 & 0.3 & 0.5\end{bmatrix}\begin{bmatrix}160\\140\\180\end{bmatrix}=\begin{bmatrix}146\\154\\180\end{bmatrix}$

Q4b $\begin{bmatrix}0.5 & 0.7 & 0.5\\0.5 & 0.3 & 0.5\\0 & 0 & 0\end{bmatrix}^6\begin{bmatrix}146\\154\\180\end{bmatrix}\approx\begin{bmatrix}280\\200\\0\end{bmatrix}$

Ali receives 280 votes to become captain.

Module 2: Networks and decision mathematics

Q1a *ABCDEFG*

Q1bi Vertex *E* Q1bii Eulerian trail

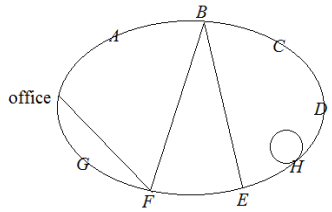
Q2a 3 activities, *H*, *I* and *J*

Q2b Critical path: *ACFI* $10+7+5+8=30$

Q2c

A	2
I	1

Q2d



Q3a Cut 1 capacity: $15+15+14+11+19=74$

Q3b Minimum cut, maximum flow: $15+10+19=44$

Q3c Reverse the flow in the pipe that runs from *Q* to *P*.

Q3d Increase the flow of the pipe that runs from *L* to *R* to at least 24 litres/minute in capacity OR from *O* to *R* to at least 28 litres/minute, so that the minimum cut (maximum flow) is 53 litres/minute.

Module 3: Geometry and measurement

Q1a $2.5(9-1.5)=18.75$ m^2

Q1b $9\times2.5\times3=67.5$ m^3

Q1c $0.85\times L=\dfrac{14.62}{8}$, $L=2.15$ m

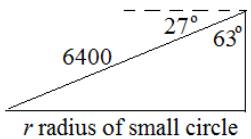
Q1d $h=\sqrt{60^2-55^2}\approx24$ m

Q1ei $60+15=75$ m

Q1eii Total area $=4\pi\times\left(\dfrac{15}{2}\right)^2+\pi\times15\times60-22.6\approx3512$ m^2

Q2a Frankfurt departure time
$=Thursday\ 6:30-9h-23h=Tuesday\ 10:30\,pm$

Q2bi $r=6400\sin63°\approx5702$ km

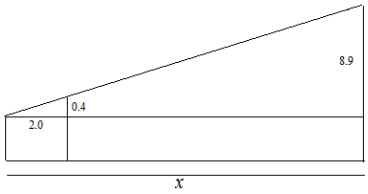


Q2bii $153-141=12$, $\dfrac{12}{360}\times2\pi\times5702\approx1194$ km

Q2c $\tan\theta=\dfrac{15}{4}$, $\theta\approx75°$

Q3a $38°-27°=11°$, distance $=\dfrac{11}{360}\times2\pi\times6400\approx1229$ km

Q3b Two similar triangles, $\dfrac{x}{8.9}=\dfrac{2.0}{0.4}$, $x=44.5$ m

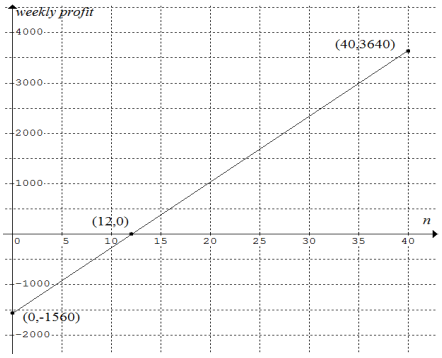


Module 4: Graphs and relations

Q1a $130\times25-1560=1690$ dollars

Q1b $130n-1560=0$, $n=12$

Q1c



Q1d $C+P=350n$, $C=350n-(130n-1560)\ \therefore\ C=220\times n+1560$

Q2a $10\times480+7\times1200=13200$ dollars

Q2b $1200q+480(25-q)=18480$, $q=9$

Q2c Use $T=420s+p$, $480\times20=420\times20+p$, $p=1200$

Q2d $k=\dfrac{F}{x}=\dfrac{500}{32}=\dfrac{800}{x}$, $x=51.2$ mm

Q3a $y\geq10$, minimum number is 10

Q3b $y=15$, maximum $x=43$

Q3c Max profit at $(38,19)$, $P_{max}=210\times38+350\times19=14630$ dollars

Q3d $(20,28)$ lies on $x+2y=76$, gradient $=-\dfrac{1}{2}$

For $(20,28)$ giving max profit, gradient of $P=px+350y$ must be greater than $-\dfrac{1}{2}$, i.e. $-\dfrac{p}{350}>-\dfrac{1}{2}$, $p<175$ dollars

Please inform mathline@itute.com re conceptual and/or mathematical errors