



Online & home tutors Registered business name: itute ABN: 96 297 924 083

2025
Specialist
Mathematics

Year 12
Problem Solving Task
(Time allowed: 2.0 hours plus)

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Problem Solving Task

Theme: Continuous random variables and probability distributions

Assumed knowledge: Functions and graphs, algebra, probability density functions, normal distributions and approximation, transformations, calculus, use of CAS

Part I (70 minutes plus)

A supermarket chain supplies apples, oranges and bananas.

The weight in grams of each fruit is a random variable and normally distributed. 99.73% (corrected to 2 decimal places) of the fruits have weights between 220 - 250 grams for apples, 200 - 230 grams for oranges, and 180 - 210 grams for bananas. Let X_a , X_o and X_b be the weights (in grams) of an apple, an orange and a banana respectively.

a. Give a reason that explains X_a , X_o and X_b are independent random variables.

b. Determine the mean and variance of each of the three fruits.

c. Given that a normal distribution is in the form $Ae^{\left(-\frac{(X-\mu)^2}{2\sigma^2}\right)}$ for $-\infty < X < \infty$, where μ and σ^2 are the mean and variance of X respectively, and A is a constant, find the normal distribution for X_a by firstly determine the value of A .

The supermarket chain pre-packs 8 apples, 12 oranges and 5 bananas for sale as an item. Let W grams be the weight of the fruits in each pre-pack.

d. Determine the mean value of W .

e. Determine the standard deviation of W .

f. Describe the distribution of W . Find $\Pr(W > 5481)$ correct to 2 decimal places.

Suppose 100 grams of apple provides 12 mg of vitamin C, 100 grams of orange provides 53 mg and 100 grams of bananas provides 9 mg.

g. Determine the expected intake of vitamin C if a shopper consumes all the fruits in a pre-pack..

There are 100 random shoppers of the supermarket chain each randomly purchasing a pre-pack weekly.

h. Determine the probability that the mean weight of the fruits in a pre-pack purchased by one of the 100 shoppers is greater than 5481 grams assuming it is normally distributed, correct to 2 decimal places.

i. Determine the probability that the orange mean weight is greater than 220 grams assuming it is normally distributed, correct to 2 decimal places.

End of Part I

Part II (70 minutes plus)

Another supermarket chain also supplies apples, oranges and bananas. Fruits in this supermarket chain are sourced from orchards different from those in Part I.

Let Y_a , Y_o and Y_b be the weights (in grams) of an apple, an orange and a banana respectively from this supermarket chain. Y_a , Y_o and Y_b are normally distributed with the following means and standard deviations.

	Mean	Standard deviation
Y_a	238	6
Y_o	220	5
Y_b	190	4

- a. Determine the 95% confidence interval for the weight of each fruit from this supermarket chain, correct to 2 decimal places.

This second supermarket chain also pre-packs 8 apples, 12 oranges and 5 bananas for sale as an item. A shopper purchases a pre-pack weekly from this supermarket chain.

Let W in grams be the weight of the fruits in a pre-pack.

- b. Determine the 95% confidence interval for the weight of the fruits in a pre-pack purchased by the shopper from this supermarket chain, correct to 2 decimal places.

There are 100 random shoppers of this supermarket chain each purchasing a pre-pack weekly.

- c. Determine the 95% confidence interval for the mean weight of the fruits in a pre-pack purchased by the 100 random shoppers assuming normal distribution, correct to 2 decimal places.

- d. Let X be a random variable in a population (in general).
Explain clearly $E(\bar{X}) = \mu$ in words using population, samples and means.

The total weights of the fruits in a pre-pack purchased by random shoppers at this second supermarket chain are taken:

5388, 5510, 5498, 5771, 5825, 5253, 5512, 5667, 5420, 5482, 5100, 5238, 5460, 5348, 5620, 5675, 5362, 5516, 5272, 5332, 5448, 5569, 5560, 5831, 5883, 5312, 5570, 5728, 5482, 5542.

- e. Find the mean value of the weight (nearest gram) of the fruits in a pre-pack purchased by the random shoppers.
Determine whether this mean value is within the 95% confidence interval for the mean weight of the fruits in a pre-pack purchased by the random shoppers assuming normal distribution.

- f. Compare the mean value of the weight of the fruits in a pre-pack in part e with the mean weight of the fruits in a pre-pack purchased by the shopper in part b.

Suppose the mean value of the weight of the fruits in a pre-pack purchased by shoppers at this supermarket chain is **unknown**, comment on a way to estimate it and how to improve the accuracy of the estimation. Determine the approximate 95% confidence interval for the weight of the fruits in a pre-pack at this supermarket chain, given the variance of the weight of the fruits in a pre-pack is 6304.

There is a third supermarket chain in the city selling the same pre-pack 8 apples, 12 oranges and 5 bananas as a single item. The management expects the mean and variance of the weight of the fruits to be the same as that of the second supermarket chain, but the suppliers to the supermarket chain say their fruits are fresher and the mean weight of the fruits in a pre-pack is greater.

The management wants to know which one of the following scenarios is more likely:

- Scenario 1 – The mean weight of the fruits in a pre-pack is the same as that of the second supermarket chain
Scenario 2 – The mean weight of the fruits in a pre-pack is greater

Pretending you as the management decides to conduct a hypothesis test.

g. State the null hypothesis and an alternative hypothesis.

h. Decide on a suitable number of random shoppers each purchasing a pre-pack at your supermarket chain for the hypothesis test.

Let the mean weight of the fruits in a pre-pack purchased by the random shoppers be 5510.

i. Calculate the p-value of the test.

j. Which hypothesis would you reject at 5% significance level? Give reasons.

End of Part II