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2027 Mathematical Methods

Year 12

Modelling Task

Time allowed: 2 hours plus

Modelling Task

Theme: Drink dispensers

Assumed knowledge: Relations and functions, piecewise functions, composite functions, sine and cosine functions, transformations, calculus, rate of change, definite integral, graphs, CAS

Information/specification:

Correct your answers to one decimal place unless stated otherwise.

Part I (60 minutes plus)

Consider $f(x) = ax^2$ and $g_1(x) = \cos(n(x-b))$ or $g_2(x) = \sin(n(x-b))$ where $a, n, b \in R$.

a. Define $h(x) = f \circ g_1(x)$ and $k(x) = f \circ g_2(x)$.

Show that $h(x) = a\cos^2(n(x-b))$. Express $k(x) = f \circ g_2(x)$ in terms of parameters a, n and b.

b. Discuss the sequence of transformations of $\cos^2 x$ caused by a, n and b in creating $h(x)$.

c. Write down the sequence of transformations required to change $h(x)$ back to $\cos^2 x$.

The rate at which the volume V (ml) of **orange** drink dispensed by a machine with respect to time t (seconds) can be modeled by $h(x) = f \circ g_1(x)$, i.e. $\frac{dV}{dt} = f \circ g_1(t)$, and the rate at which the volume V (ml) of **lemon** drink dispensed by the machine with respect to time t (seconds) can be modeled by $k(x) = f \circ g_2(x)$, i.e. $\frac{dV}{dt} = f \circ g_2(t)$.

d. For each type of drink dispensed by the machine, express $\frac{dV}{dt}$ as a function of t where $0 \le t \le 5$, given that $a = 141.0$, $n = 0.63$ and $b = 2.5$ for orange, and $a = 143.2$, $n = 0.64$ and $b = -5.0$ for lemon.

For the following questions select $\frac{dV}{dt}$ for only one type of drink, orange or lemon, NOT both.

e. Determine the maximum rate in dispensing drink by the machine and the time when it occurs.

f. Determine the volume of drink (ml) dispensed by the machine each time.

g. Determine by calculation (NOT by CAS) the magnitude of the maximum rate of change in the rate $\frac{dV}{dt}$, and the value(s) of t (seconds, correct to 3 decimal places) when it occurs. Include units in your answers.

Hint: Use $2\sin\theta\cos\theta = \sin(2\theta)$ to simplify your calculation.

h. The shopkeeper wants to decrease the volume of drink dispensed by the machine by 2% each time but keep the dispensing time constant at 5 seconds.

Which one of parameters a, n and b in the mathematical model for $\frac{dV}{dt}$ would you change to decrease the volume by 2%? Explain your choice and why not the other two parameters.

i. Determine the value of your chosen parameter in part h, which will give a 2% decrease in the volume of drink each time.

End of Part I

Information/specification:

Correct your answers to two decimal places unless stated otherwise.

Part II (60 minutes plus) Volume V in milli-litres (ml); time t in seconds (s)

The coffee machine is programmed to prepared a 300 ml of white coffee (plain coffee with added milk). The ratio of added milk to plain coffee is $1:9$.

When the coffee machine is turned on, milk is first added to an empty cup at a rate given by

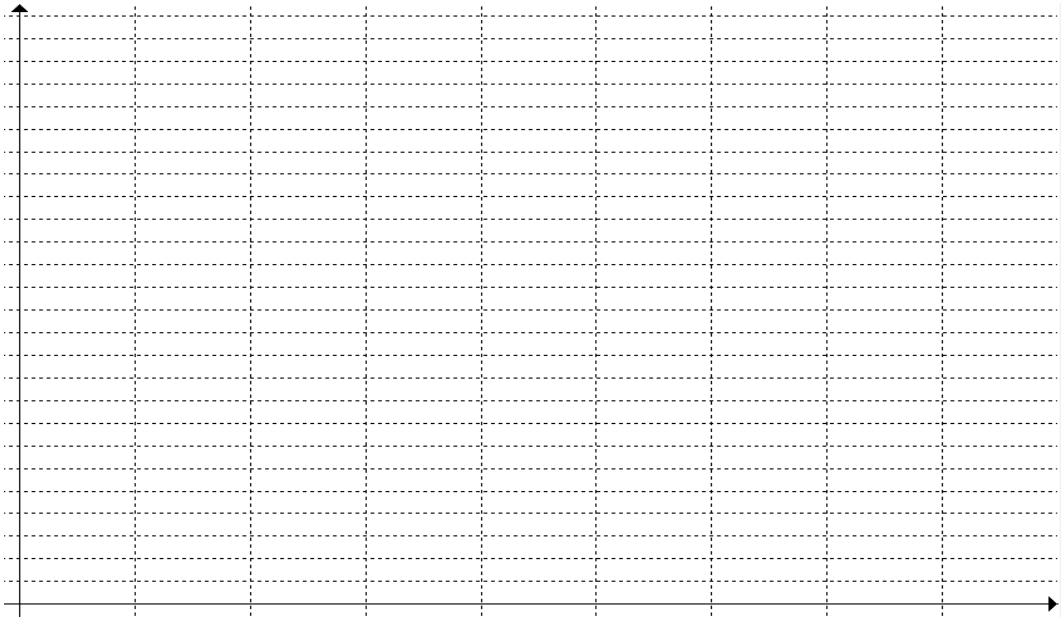
$$\frac{dV}{dt} = 3\left[\sin\frac{\pi}{5}\left(t-\frac{5}{2}\right)+1\right] \text{ for } 0 \le t \le 10.$$

At $t = 5$ plain coffee starts to run into the cup (while milk is dispensing) at a rate given by

$$\frac{dV}{dt} = 9\left[\sin\frac{\pi}{15}\left(t-\frac{15}{2}\right)+1\right] \text{ for } 5 \le t \le 35.$$

a. State a sequence of transformations on $\frac{dV}{dt}$ for added milk required to change it to $\frac{dV}{dt}$ for plain coffee.

b. Sketch the graph of $\frac{dV}{dt}$ for each rate against t on the same set of axes. Label each graph clearly with MILK or COFFEE. Make full use of the provided space.



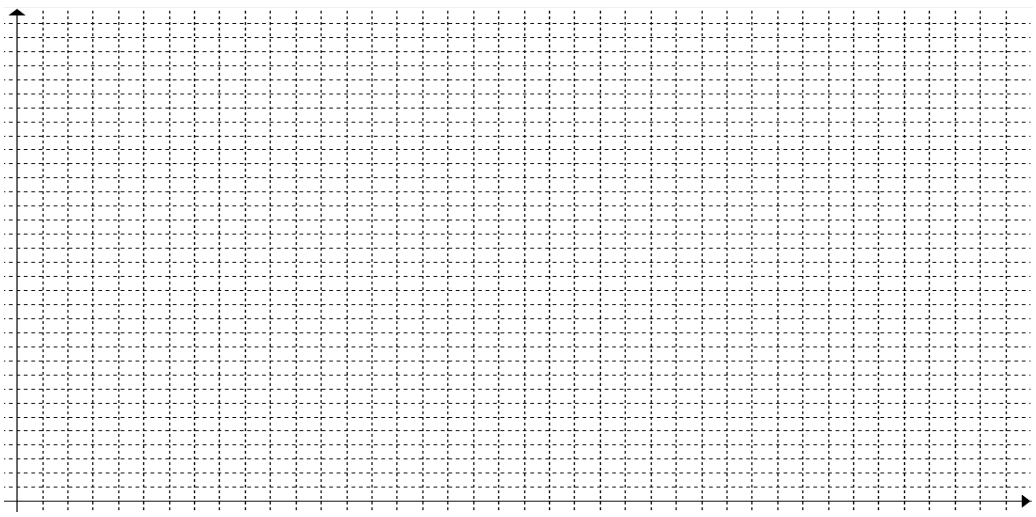
c. Let $W(t)$ be the total volume of added milk and plain coffee in the cup at time t. On the above set of axes, use the method of addition of ordinates to sketch the graph of $\frac{dW}{dt}$ against t for the time when liquid is filling the cup.

d. Write a piecewise function of t to model $\frac{dW}{dt}$ for $t \geq 0$.

e. Determine the amount (ml) of liquid enetering the cup in the first 5 s.

f. State the time interval(s) when $\frac{dW}{dt}$ is strictly increasing.

g Write a piecewise function of t to model W for $t \geq 0$. With the help of CAS sketch the graph of W against t.



The shopkeeper purchased a new machine which can make the same cup of white coffee faster at 60% of the time taken by the original machine. You are allowed to use sine function, cosine function, or function of your choosing for the next two parts.

h. The machine is programmed such that plain coffee starts to dispense when milk stops dispensing.

Write down a possible $\frac{dV}{dt}$ for added milk and $\frac{dV}{dt}$ for plain coffee.

i. The machine is programmed such that plain coffee starts to dispense while milk is still dispensing.

Write down a possible $\frac{dV}{dt}$ for added milk and $\frac{dV}{dt}$ for plain coffee.

End of Part II

End of Modelling Task